Reconnection and dynamo

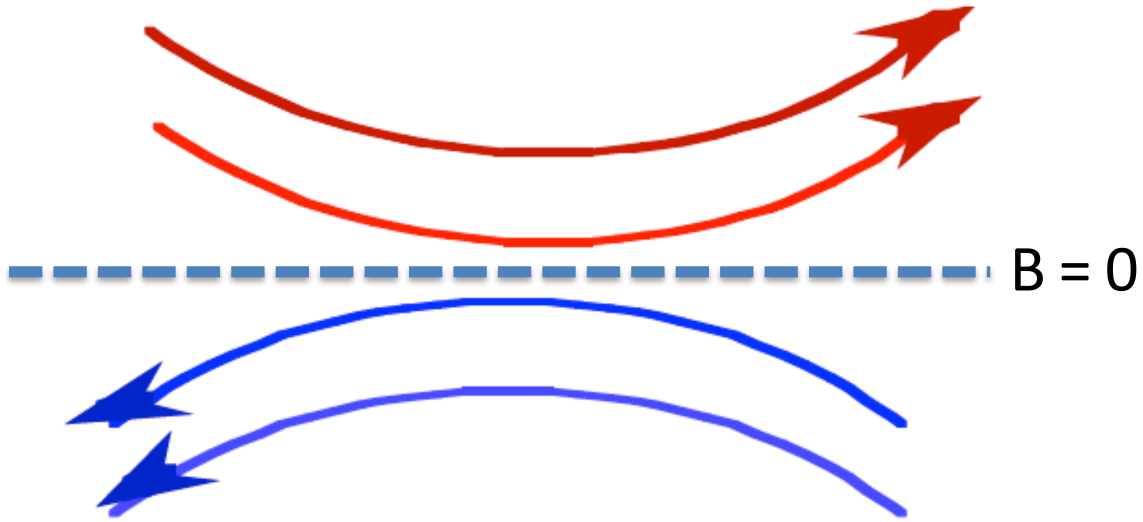
Fatima Ebrahimi and Stewart Prager

Lecture plan

1.	Reconnection basics	SP
2.	Dynamo basics	FE
3.	Application to lab plasmas	SP
4.	Application to space/astrophysical plasmas	FE
5.	Challenges and open questions	FE (probably)

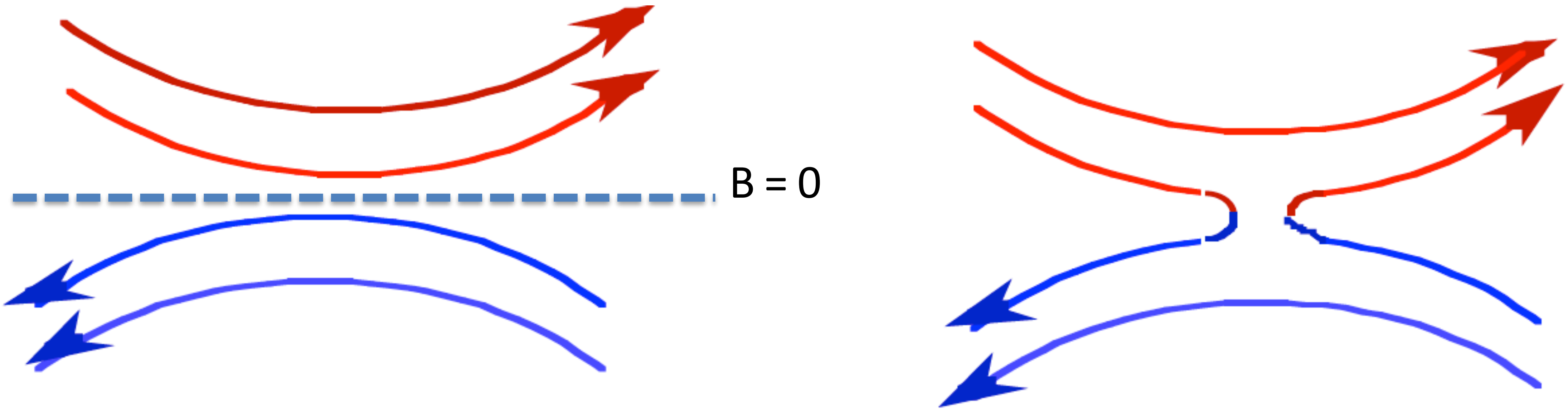
Magnetic Reconnection Basics

Consider magnetic field with a null line



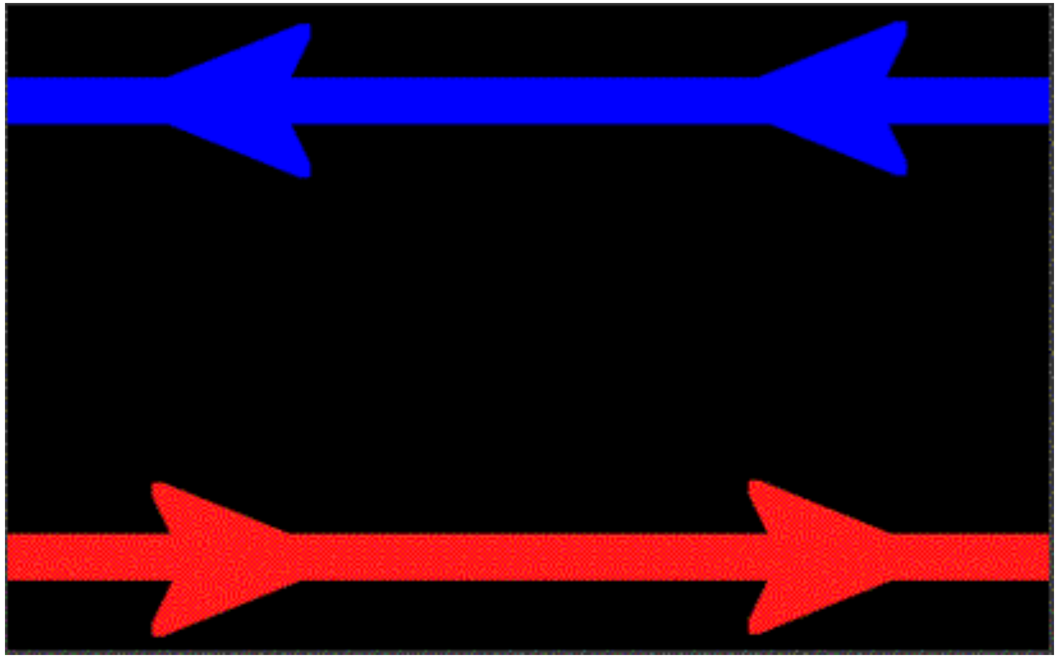
Before reconnection

Consider magnetic field with a null line



Before reconnection After reconnection

This is magnetic reconnection

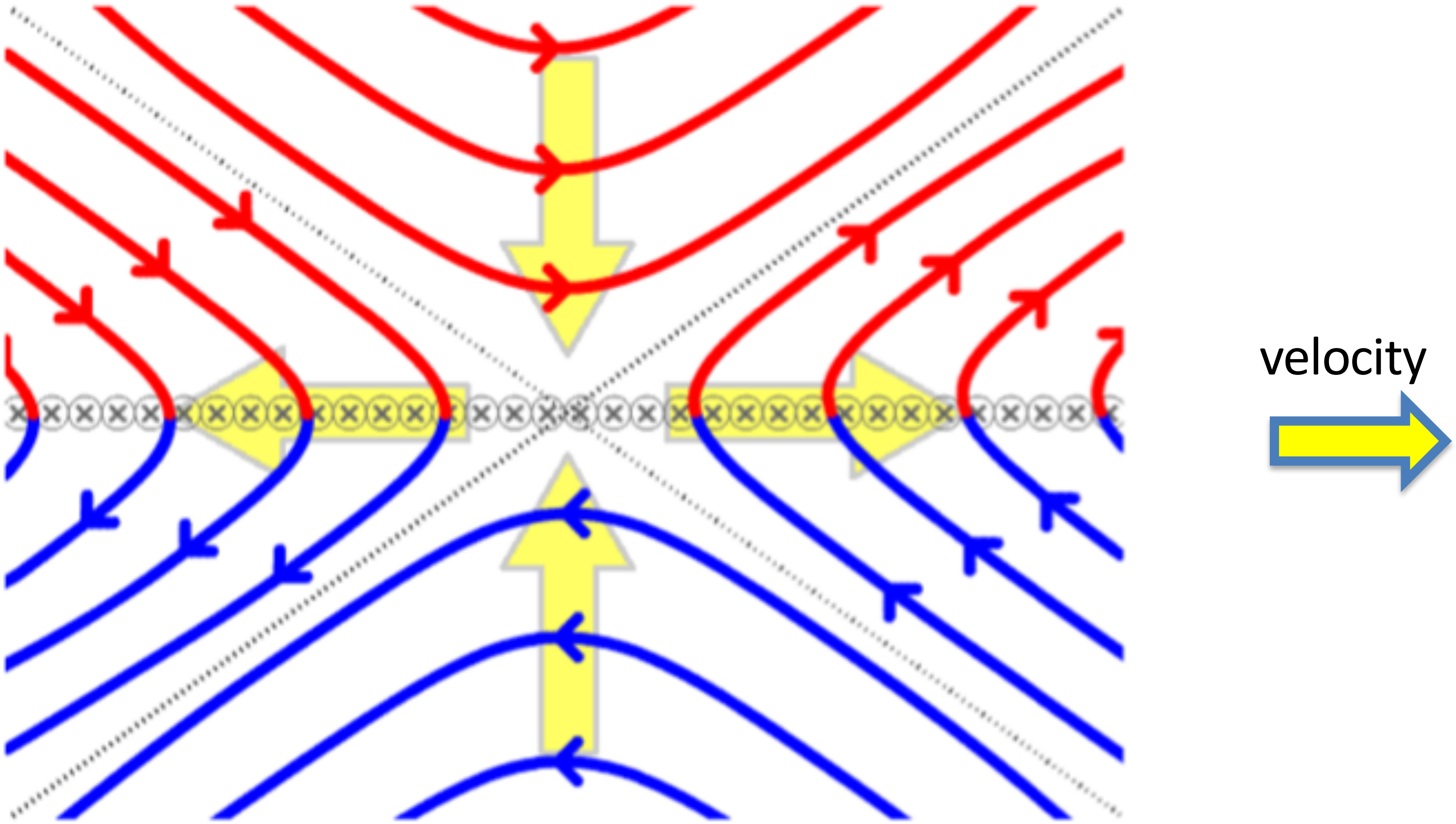


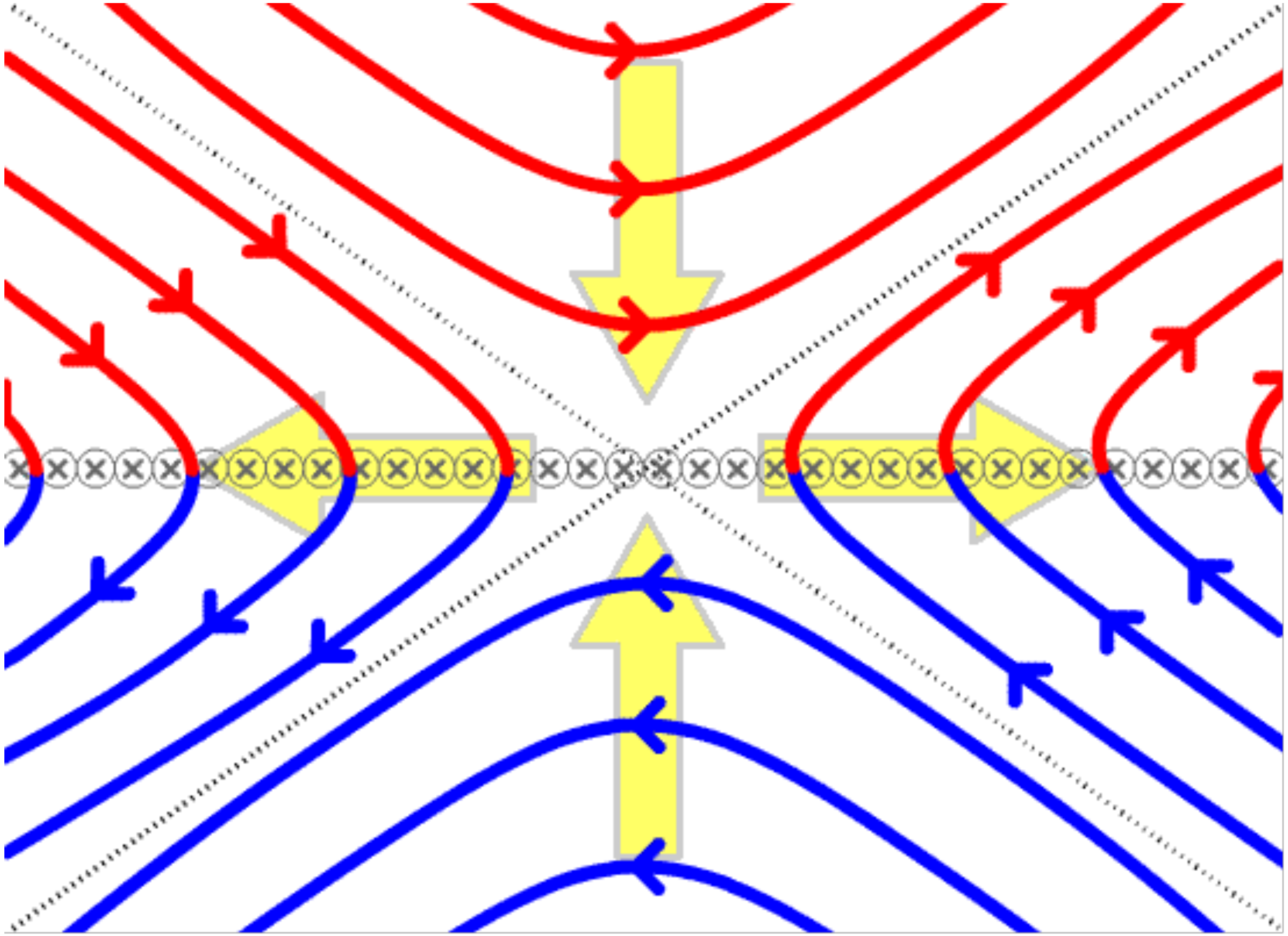
Magnetic field lines tear and reconnect

Can be forced or spontaneous

Magnetic field annihilation

Reconnection can also occur at a null-point (x-point)





Why is reconnection important?

Drives plasma transport

- of particles, energy, momentum
- e.g., particles tied to B lines that can change drastically

Heats plasma and accelerates particles

- e.g., sudden changes in B implies strong E field

Reconnection: magnetic energy ⇒ kinetic energy

Dynamo: kinetic energy (plasma flow) ⇒ magnetic energy

and,

reconnection often accompanies or underlies dynamo processes

Where is reconnection important?

Seems like almost everywhere plasmas are magnetized

In the cosmos:

Geomagnetic storms (space weather),

solar activity: flares, coronal mass ejections…

star formation

flares in accretion disks, magnetars……

heating and acceleration (cosmic rays, pulsars, astrophysical jets……)

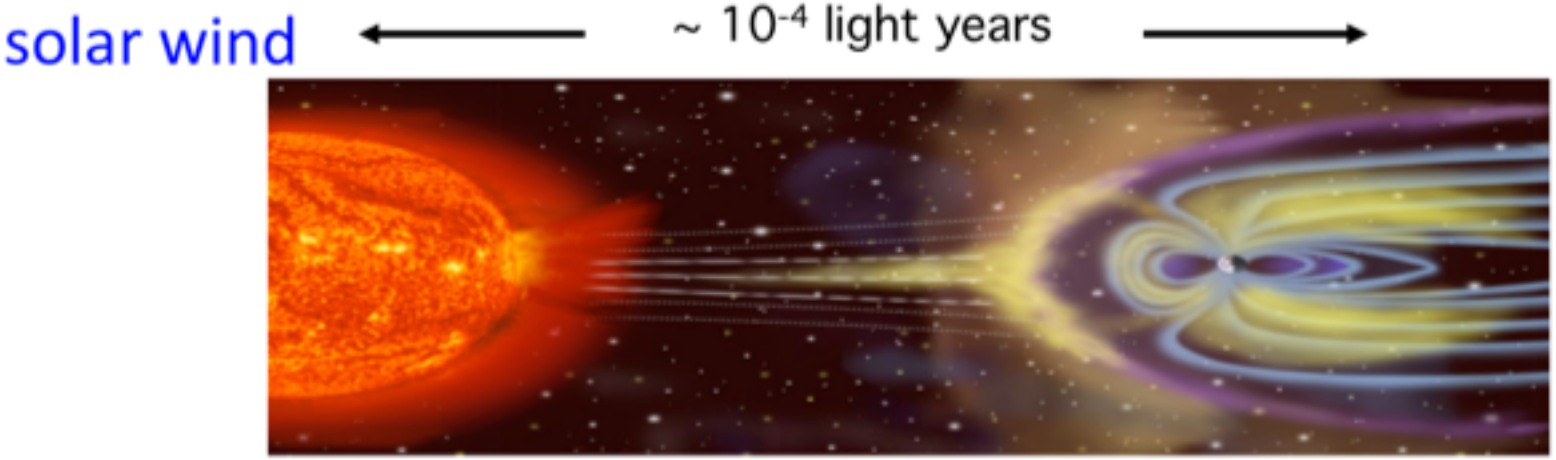
In fusion plasmas:

tokamak sawtooth oscillations and disruptions

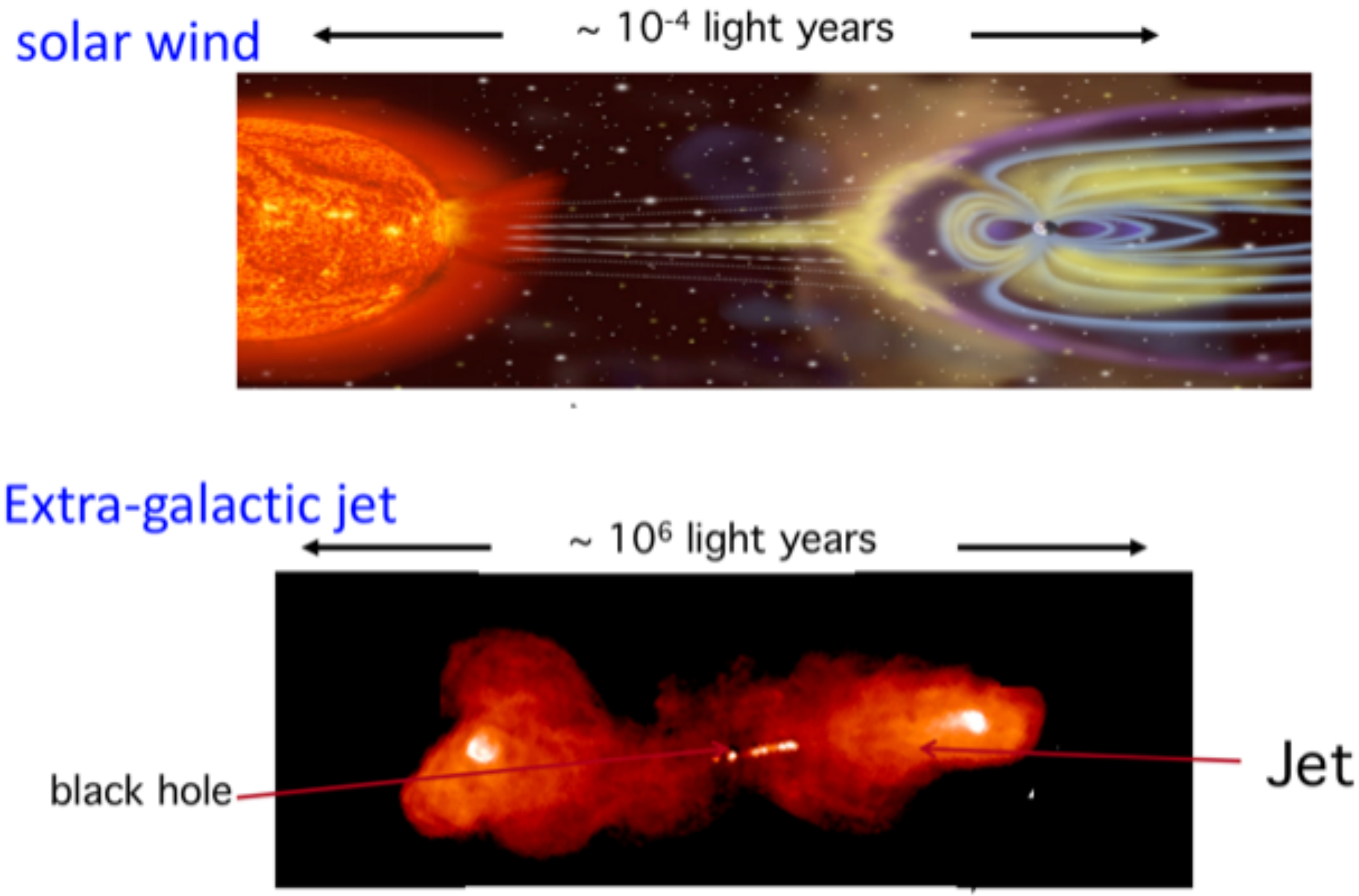
overall behavior of plasmas with safety factor < 1 (weak B field)

helicity injection current drive

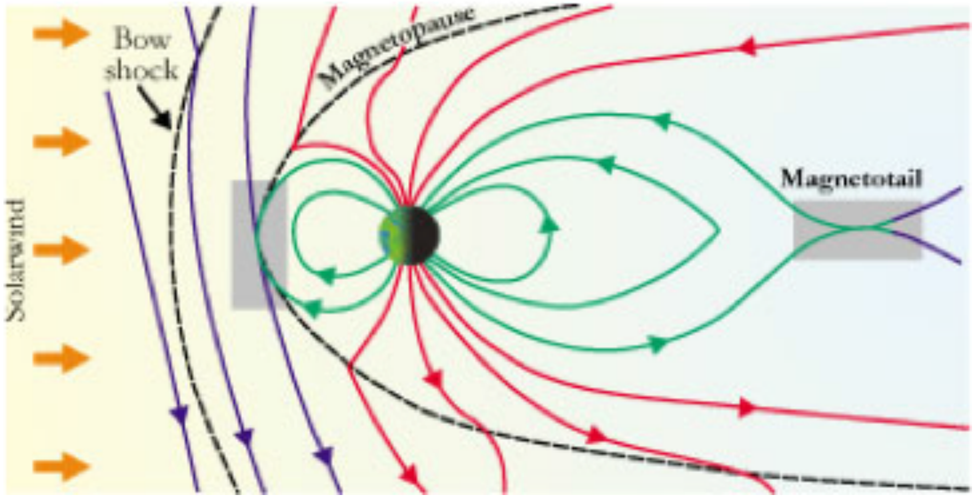
Astrophysical plasmas and reconnection span enormous scales



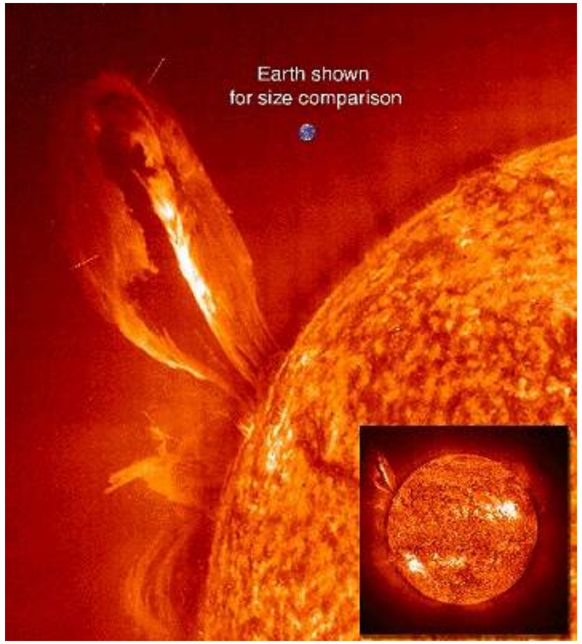
Astrophysical plasmas and reconnection span enormous scales



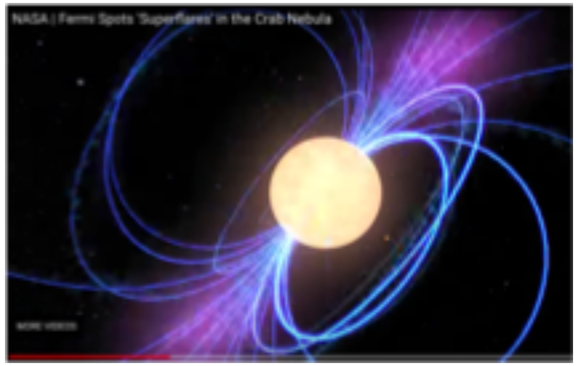
Earth's magnetosphere and geomagnetic storms



Solar flares

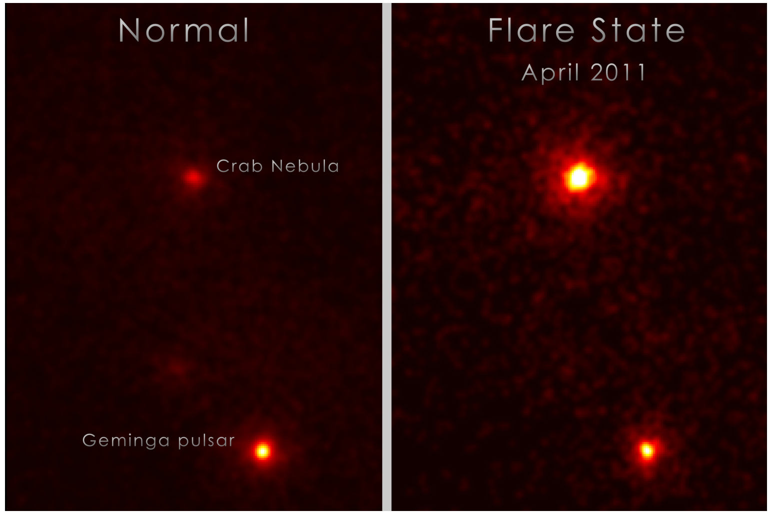


Gamma ray flares from crab nebula



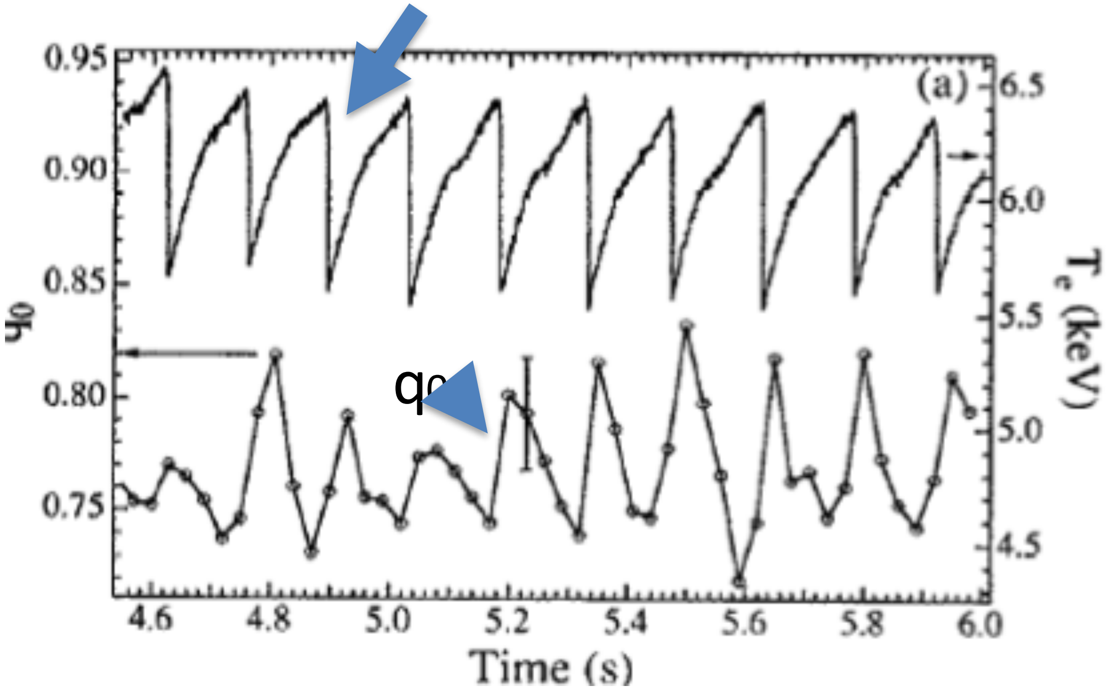
Rotating neutron star

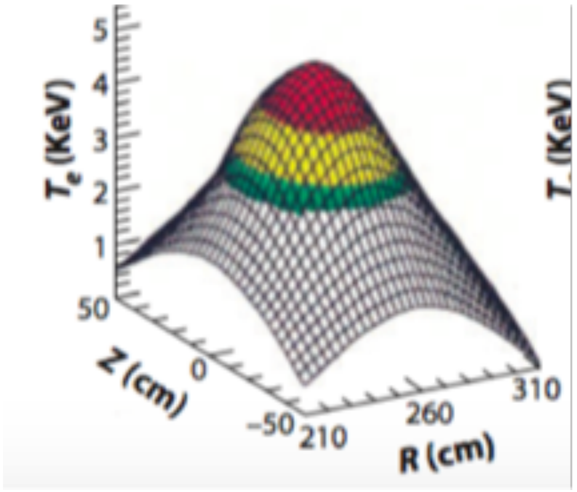
Gamma ray flare:
electron acceleration

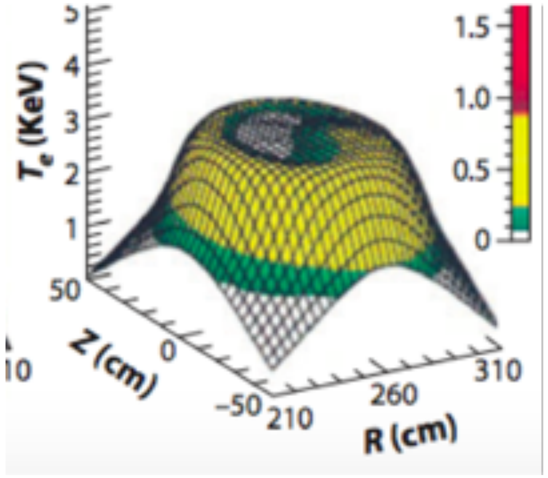


Sawteeth (relaxation oscillations) in tokamaks

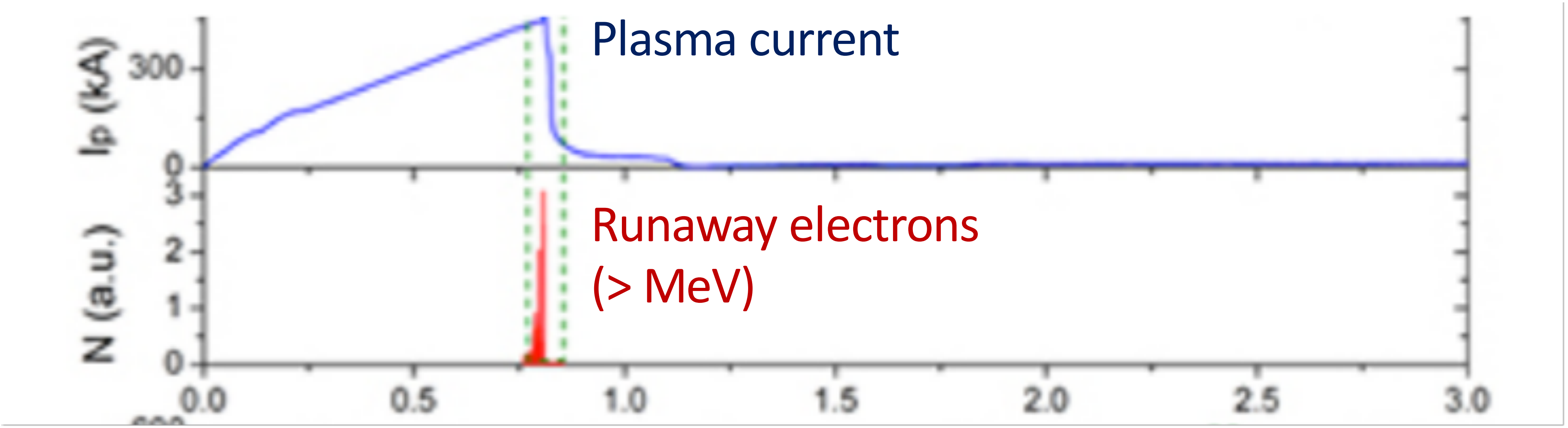
Electron temperature







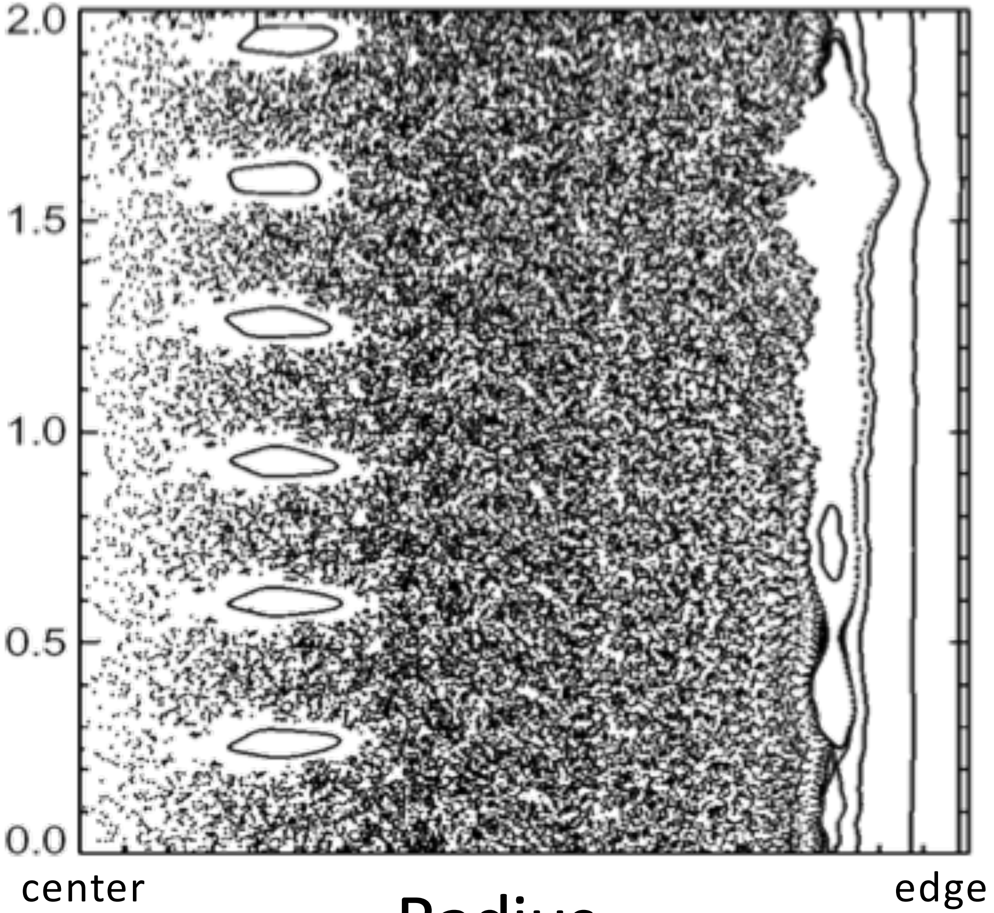
Disruptions in tokamaks



Time (sec)

Reconnection can produce a chaotic field

Toroidal angle



Radius

MHD computation
For torus with weak field

Why is reconnection a physics challenge?

- Two coupled regions of disparate scales
  - Microscopic region in the vicinity of reconnection
    - intricate physics
    - phenomena important, from MHD to kinetic
  - Macroscopic region
    - the full plasma, affected by behavior in microscopic region
- Reconnection drastically affects plasma through many mechanisms

Many (coupled) fundamental questions

For example

- Why is reconnection fast?
- Does the reconnection layer break up into plasmoids?
- How does reconnection provide acceleration and heating?
- How does reconnection behave in partially ionized plasmas?
- Why does reconnection often onset suddenly?

Requires theoretical treatment from MHD to kinetic theory

Reconnection cannot occur in an ideal plasma

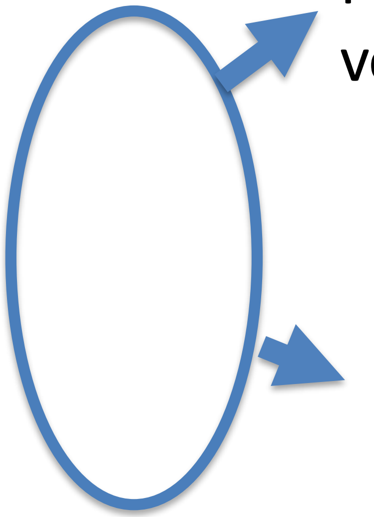
- Ideal = perfectly conducting

- The magnetic field is frozen into the plasma fluid

  the field cannot ”tear”

The frozen flux theorem

Consider a loop moving with the plasma



The rate of change of magnetic flux within the loop is

$$\frac{d\Phi}{dt} = \oint \boldsymbol{E'} \cdot \boldsymbol{dl}$$

where Φ = magnetic flux through moving loop

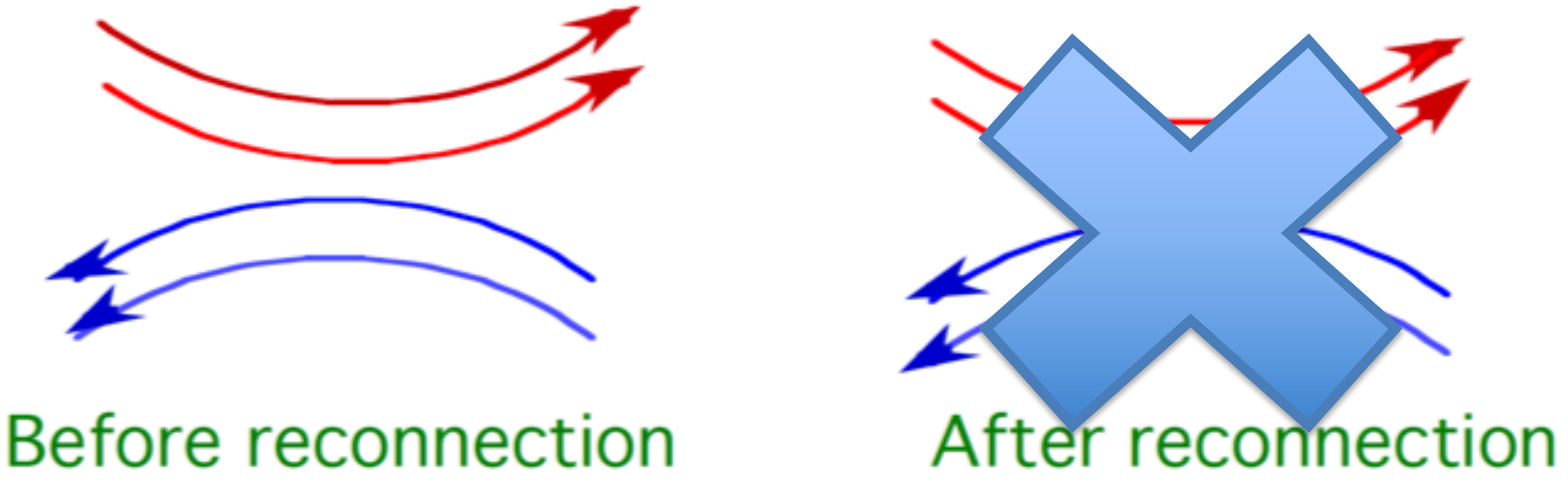
$\boldsymbol{E'}$ = electric field in loop frame = $\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B}$

$= \eta \boldsymbol{j}$ by Ohm's law

$= 0$ for an ideal plasma

Therefore $$\frac{d\Phi}{dt} = 0$$ Magnetic field is frozen into plasma

Thus, reconnection cannot occur in an ideal plasma



i.e., can show that if reconnection occurs, the plasma flow velocity must be infinite

Reconnection can occur in a resistive plasma

A dimensionless measure of resistivity

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}$$

$$= \nabla \times (\vec{v} \times \vec{B} - \eta \vec{j}) = \nabla \times (\vec{v} \times \vec{B} - \eta \frac{\nabla \times \vec{B}}{\mu_0})$$

Ideal resistive

Dimensionless measure of resistivity $S = \frac{ideal}{resistive} = \mu_0 \frac{vL}{\eta}$

let $v = v_A$ where $v_A = \frac{B}{\sqrt{\mu_0 \rho}}$ Alfven speed,

define

$\tau_A = \frac{L}{v_A}$ Alfven time, $\tau_R = \frac{\mu_0 L^2}{\eta}$ Resistive diffusion time

A dimensionless measure of resistivity

then

$$S = \frac{\tau_R}{\tau_A}$$

Lundquist number

A wide range of S values in nature $S \sim \frac{T^{3/2} L^3 B}{n^{1/2}}$

Lab		**Galaxy**	
Basic plasma experiments	$10 - 10^4$	Protostellar disks	10^4
Fusion experi			10^{16}
			10^{20}
Solar system			
Geomagnetic tail	10^{15}	AGN disks	10^{13}
Solar wind	10^{12}	AGN disk coronae	10^{23}
Solar corona	10^{14}	Jets	10^{29}

S tends to be large

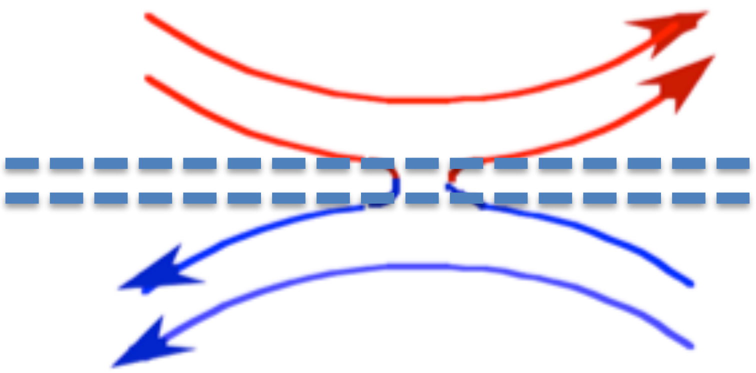
Then why is resistivity important?

- Permits reconnection even though S is huge

- Mathematically, why is resistivity important if terms with resistivity is small?

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times \left(\vec{v} \times \vec{B} - \eta \frac{\nabla \times \vec{B}}{\mu_0}\right)$$

$$\sim \eta \nabla^2 \vec{B}$$

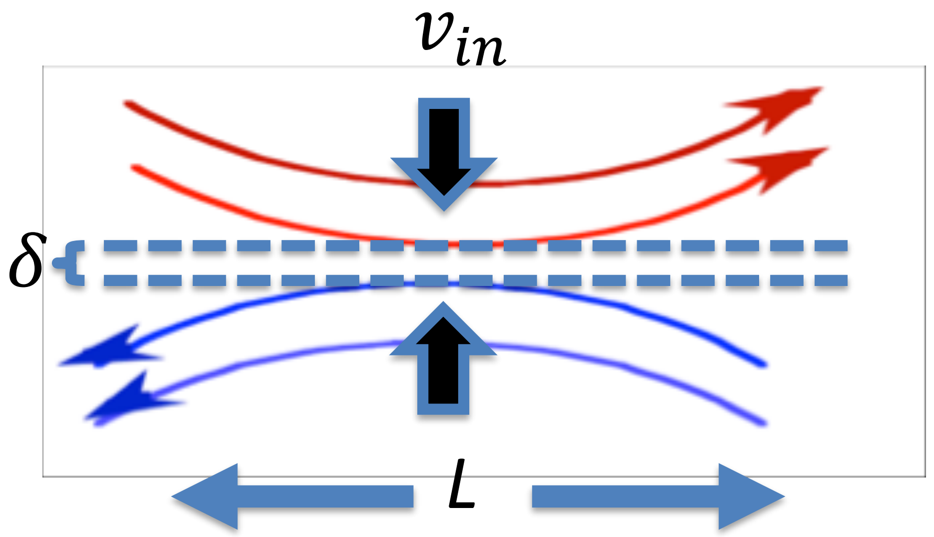


Large in reconnection layer

A simple MHD dimensional analysis
(Sweet-Parker Model, 1958)

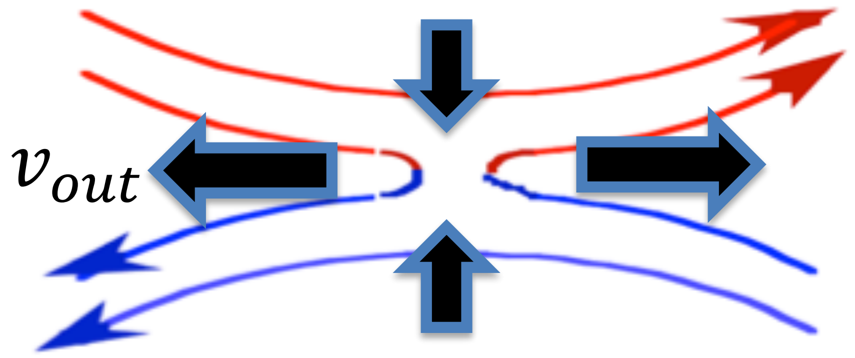
δ = reconnection layer width (or resistive layer width)

v_{in} = inflow velocity, v_{out} = outflow velocity



Mass conservation $v_{in} L \sim v_{out} \delta$

E x B drift $v_{in} \sim \frac{E_Z}{B} \sim \frac{\eta j}{\mu_0 j \delta} \sim \frac{\eta}{\mu_0 \delta}$



Mag energy in = KE out $\frac{B^2}{2\mu_0} \sim \frac{\rho v_{out}^2}{2}$ → $v_{out} = v_{Alfven}$

Merging 3 equations

$$\frac{v_{in}}{v_{out}} \sim S^{-1/2}$$

Dimensionless reconnection rate

Dimensional Reconnection time

$$\tau_{rec} \sim \frac{B}{\partial B / \partial t} \sim \frac{B}{E / L} \sim \frac{1}{v_{in} L} \sim \frac{v_{Alfven}}{L} S^{1/2}$$

$$\tau_{rec} \sim S^{1/2}$$

"Sweet Parker reconnection"

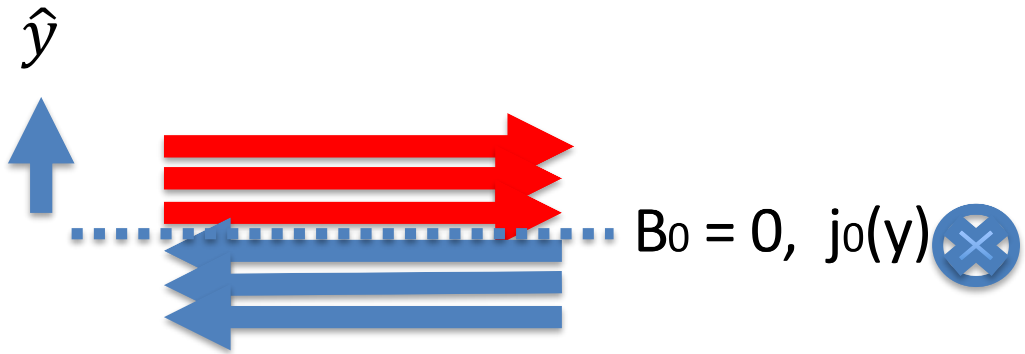
Reconnection time becomes large at very high S (conductivity); i.e., reconnection is weak at high S

Also, reconnection layer width $\delta \sim L \frac{v_{in}}{v_{out}} \sim L S^{-1/2}$ *Small at high S*

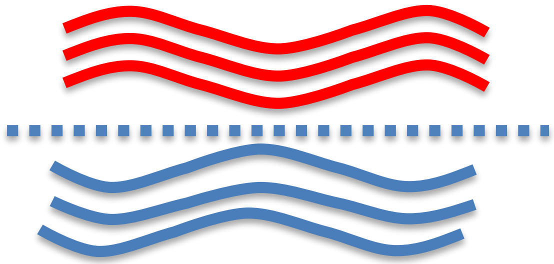
A rigorous linear analysis of spontaneous reconnection

Reconnection from an MHD instability generated by current density gradient
("tearing instability" – Furth, Killeen, Rosenbluth, 1963)

Equilibrium:



Perturbation:



All perturbed quantities of the form

$$\tilde{B}, \tilde{v} \sim f(y) \sin(kx + \theta) e^{\gamma t}$$

k = wave number
γ = growth rate

Basic equations:

$$\rho \frac{\partial \vec{v}}{\partial t} = \vec{j} \times \vec{B} - \nabla p$$

Taking $\nabla \times$

$$\rho \nabla \times \frac{\partial \vec{v}}{\partial t} = \nabla \times (\vec{j} \times \vec{B})$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B} - \eta \frac{\nabla \times \vec{B}}{\mu_0})$$

write $B = B_0(\vec{r}) + \tilde{B}(\vec{r}, t)$, with $\tilde{B} << B_0$, and linearize equations

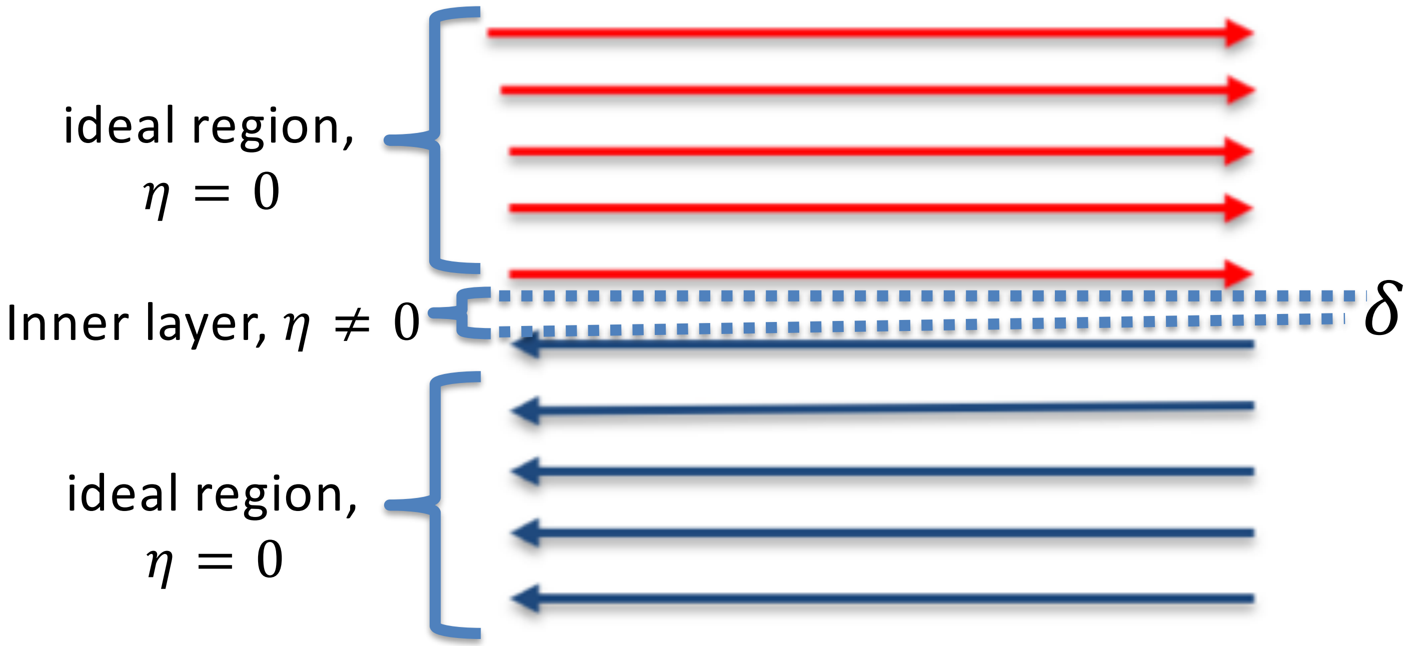
4th order system to solve for $\vec{v}(y)$, $\vec{B}(y)$ and $\gamma(k)$

How to solve analytically? (gives large insight)

Separate plasma into two regions

Resistive inner layer
Small region where reconnection occurs
Resistivity is important

Ideal outer regions
Resistivity set to zero
resistivity is unimportant if S is large
Soluble 2nd order equation



Match two solutions at boundary
Boundary layer analysis, or singular perturbation theory

How to solve inner layer equations?

Simplify inner layer equations by small parameter expansion

Assume an ordering of all quantities in small parameter ε

e.g.,

$\delta \sim \varepsilon$	thin resistive tearing layer
$S \sim \varepsilon^{-5/2}$	defines ε as small resistive parameter
$\frac{\partial}{\partial y} \sim \varepsilon^{-1}$	strong gradients of perturbation in layer
$B_0 \sim \varepsilon$	weak equilibrium field

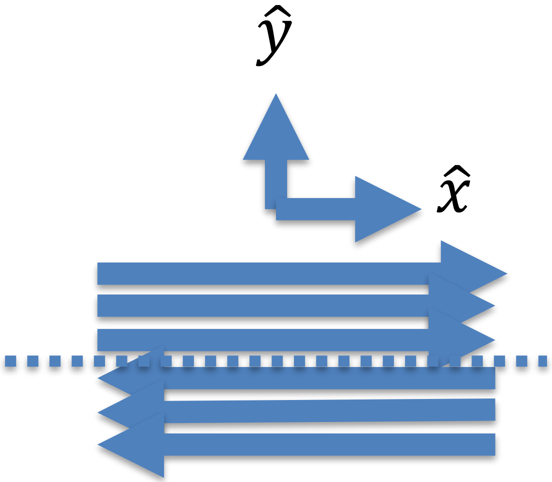
Will not solve here, but only display form of solutions

First examine ideal solution

The structure of the solution to ideal equations

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \bar{E})$$

$$= -(\vec{v} \cdot \nabla)\vec{B} + (\vec{B} \cdot \nabla)\vec{v}$$



Recall,

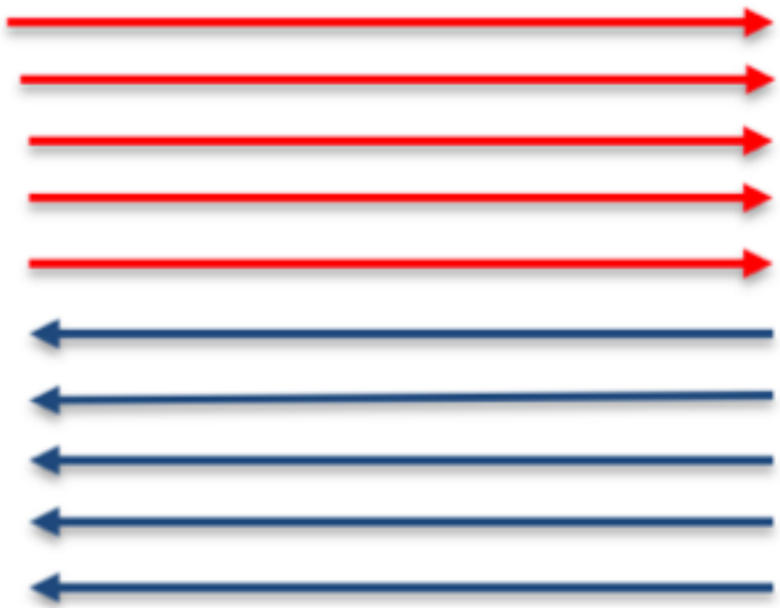
$$\vec{B} = \vec{B}_0 + \tilde{\boldsymbol{B}}, \quad \vec{v} = \tilde{\boldsymbol{v}}$$

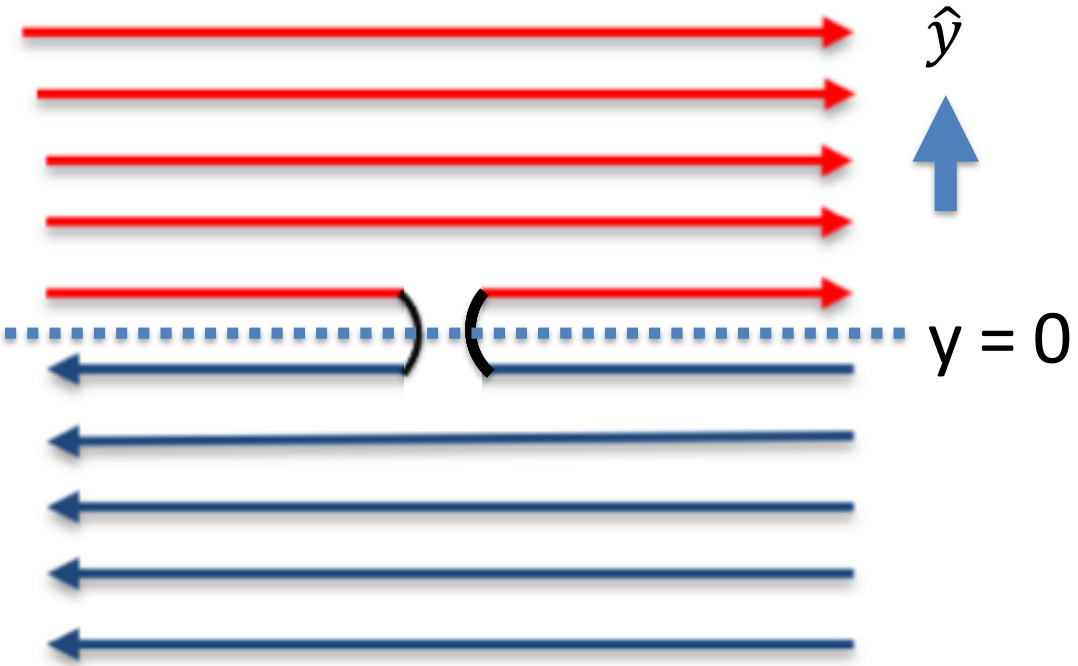
$\tilde{B}, \tilde{\boldsymbol{v}} \sim e^{(\gamma t + ikx)}$ Small quantiities

Linearizing the y-component of the above equation,

$$\gamma \widetilde{B_y} = ikB_0 \widetilde{v_y}$$

Note: Reconnection requires $\widetilde{B_y} \neq 0$ at y = 0





$$\gamma\widetilde{B_y} = ikB_0\widetilde{v_y}$$

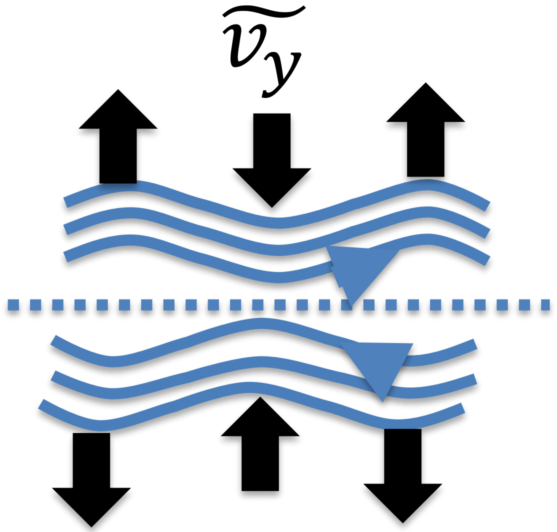
Examine solution near reconnection layer (y = 0) where

$$B_0 \sim y = 0$$

then
$$\widetilde{v_y} \sim i\frac{\widetilde{B_y}}{y}$$

Therefore, at $y = 0$, $\widetilde{B_y} = 0$ (or else $\widetilde{v_y}$ diverges)

no reconnection in an ideal plasma

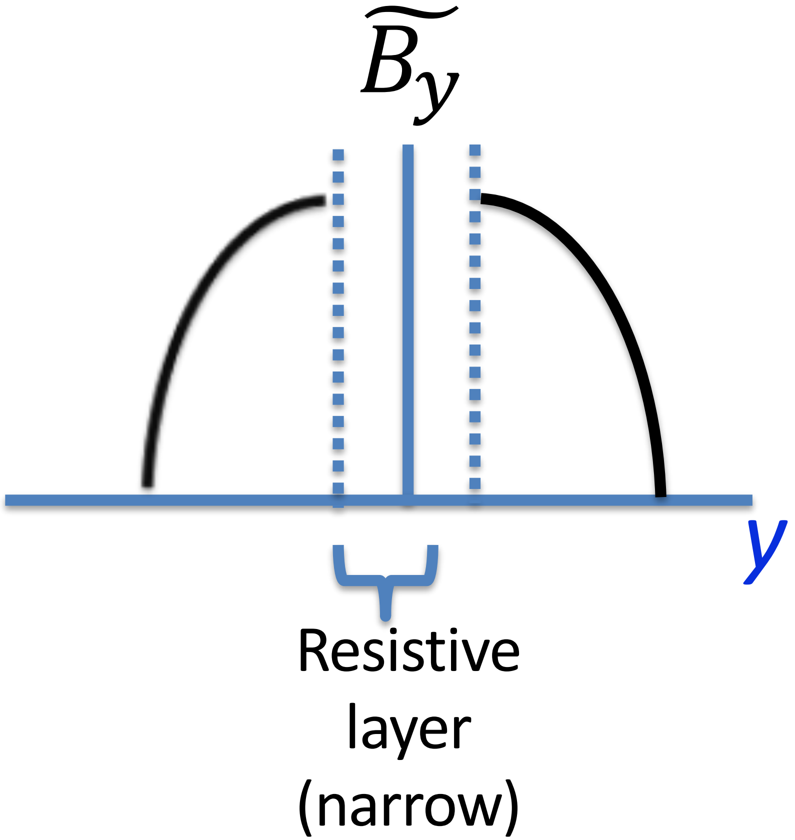


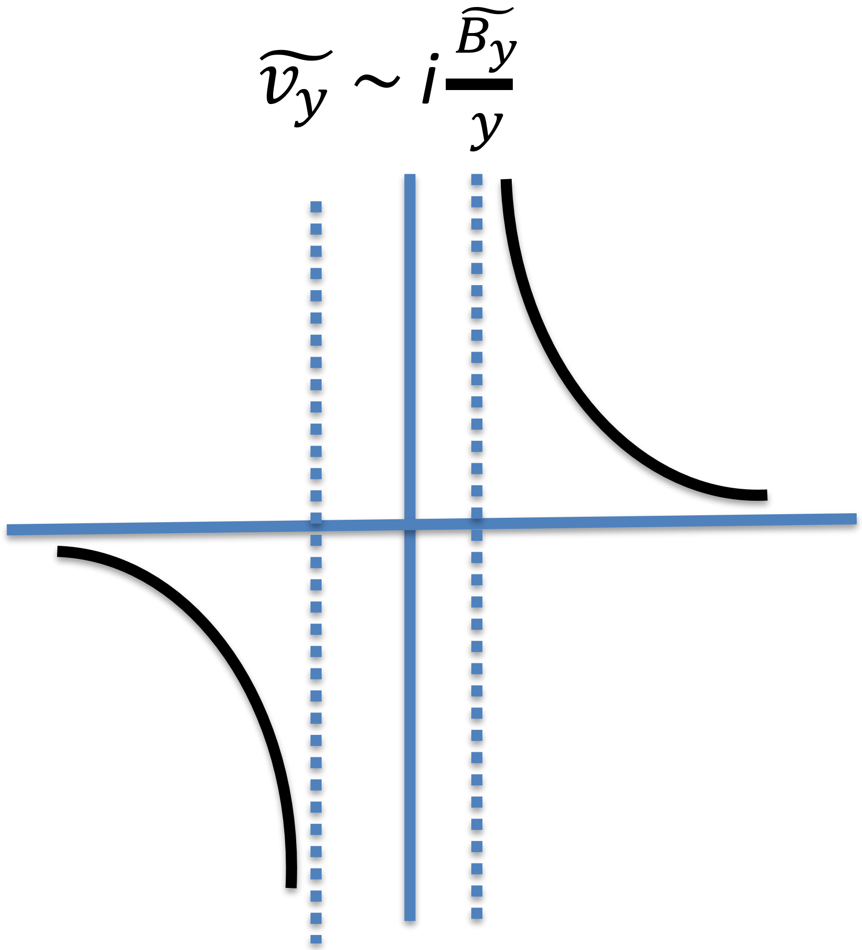
$\widetilde{v_y}$ and $\widetilde{B_y}$ are out of phase

$\widetilde{v_y}$ is asymmetric, $\widetilde{B_y}$ is symmetric

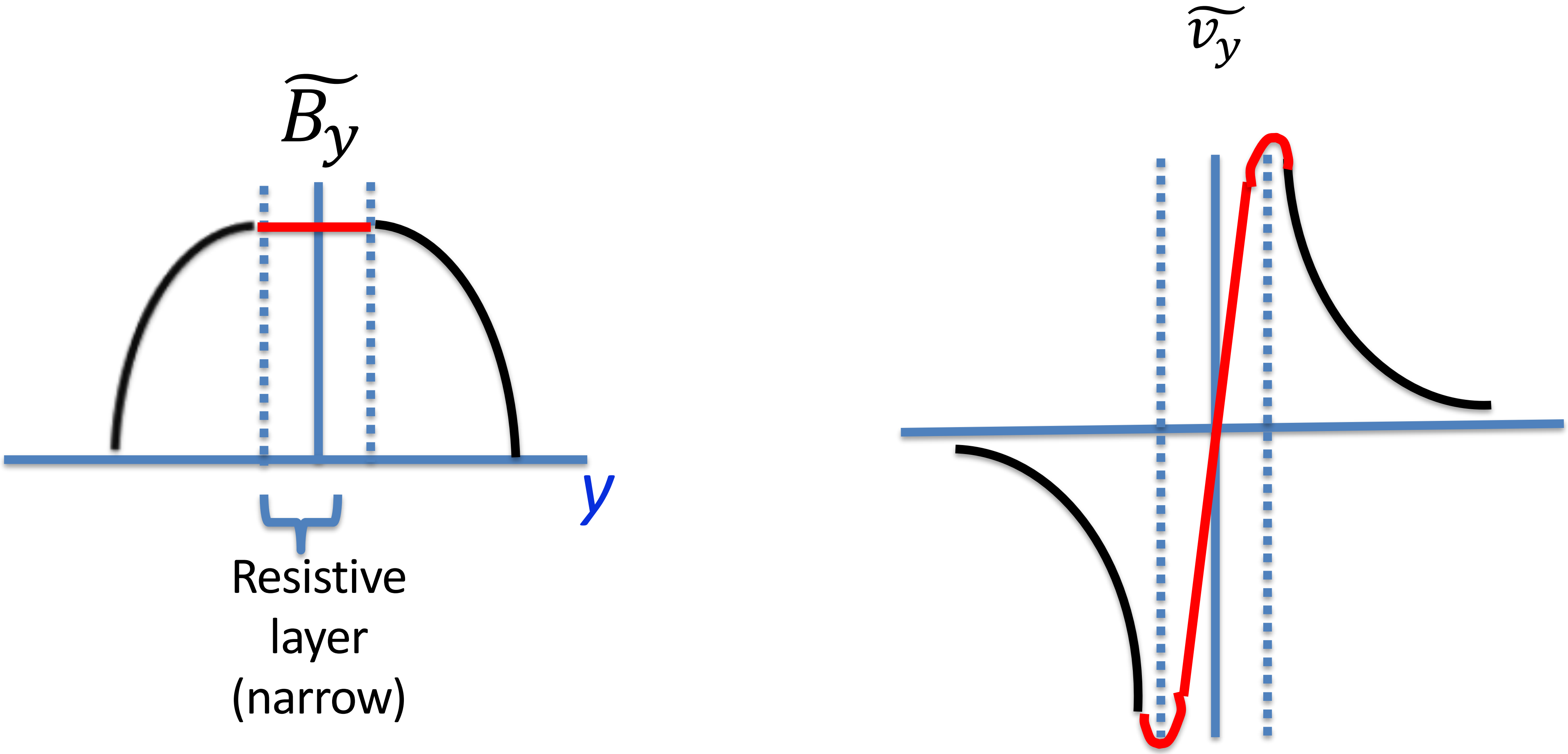
Ideal solutions

we are only applying ideal solutions near y = 0, not at y = 0





Solutions in resistive layer



Resistive layer width $\delta \sim S^{-2/5}$ very small at high S

(vs. Sweet Parker $\delta \sim S^{-1/2}$)

Growth time (γ^{-1}) $\tau_g \sim \tau_A S^{3/5}$ large at high S

(vs SP, $\tau_{rec} \sim S^{1/2}$)

or

$\tau_g \sim {\tau_R}^{3/5} \, {\tau_A}^{2/5}$ hybrid timescale

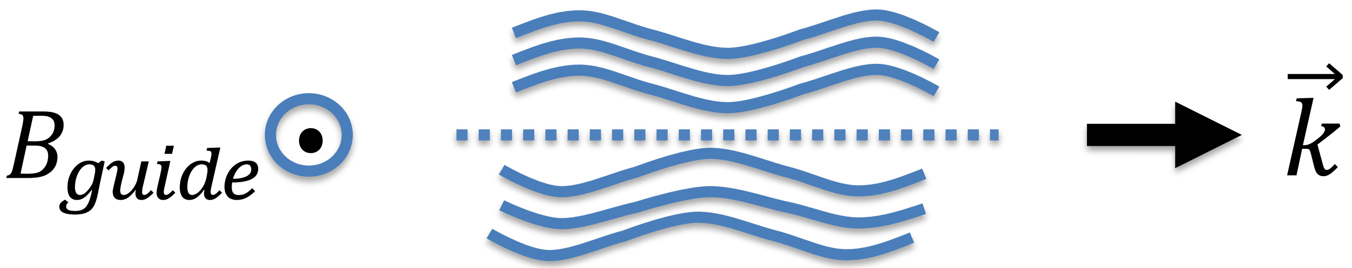
$\tau_A \ll \tau_g \ll \tau_R$

This instability is called the tearing instability (spontaneous reconnection)

Stability depends upon the strength of the current density gradient

Can reconnection occur with B is nonzero everywhere?

Yes, can add a straight magnetic field out of the plane (sometimes called a "guide field"), without altering the above arguments



Then, reconnection occurs where, and if, there is a location where

$$\vec{B} \cdot \vec{k} = 0$$

The above resistive MHD models provide enormous insight, but they have severe limitations

First limitation: Fails to predict reconnection timescales

Solar flare: observed reconnection time ~ 15 min – 1 hr

resistive diffusion time, $\tau_R \sim 1\, Myrs$

SP or tearing recon time, $\tau_{SP} \sim 2\, months$ *still too slow!*

Sawtooth oscillations in tokamak:

observed crash time < 0.5 msec

resistive diffusion time, τ_R ~10 sec

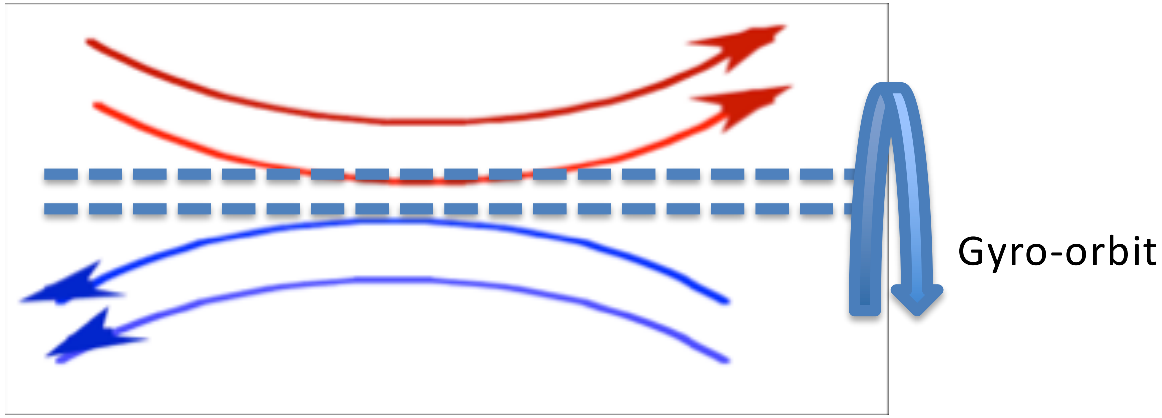
SP or tearing recon time, $\tau_{SP} \sim 100\, ms$ *still too slow!*

Second limitation: kinetic effects

e.g, at high S, gyroradius >> reconnection layer width

in fusion plasma: $\delta_{SP} < 1\,mm$

ion gyroradius ~ 1 mm – 1 cm



Third limitation: nonlinear effects

Fourth limitation: treatment of effects of reconnection on plasma (transport, heating……)

A two-fluid description

Consider fluid momentum equations for each species

$$m_i n_i\left(\frac{\partial \mathbf{V}_i}{\partial t}+(\mathbf{V}_i\cdot\nabla)\mathbf{V}_i\right)=en_i(\mathbf{E}+\mathbf{V}_i\times\mathbf{B})-\nabla\cdot\mathbf{P}_i-\mathbf{R}$$

$$m_e n_e\left(\frac{\partial \mathbf{V}_e}{\partial t}+(\mathbf{V}_e\cdot\nabla)\mathbf{V}_e\right)=-en_e(\mathbf{E}+\mathbf{V}_e\times\mathbf{B})-\nabla\cdot\mathbf{P}_e+\mathbf{R}$$

where $\boldsymbol{P_i}$ and $\boldsymbol{P_e}$ are pressure tensors $P_{ij}(\vec{x}, t) = \int m(v_i - V_i)(v_j - V_j)\, f(\vec{x}, \vec{v}, t) d^3 v$

R is the friction force between electrons and ions

Combining the two equations yields the Generalized Ohm's law

Generalized Ohm's Law

$$\vec{E} + \vec{v} \times \vec{B} = \eta \vec{j} + \frac{\vec{j} \times \vec{B}}{ne} - \frac{1}{ne} \nabla \cdot \mathbf{P}_e - \frac{m_e}{e} \frac{dv_e}{dt}$$

Simple Ohm's law

Hall term

Electron pressure tensor

Electron inertia

Does NOT break frozen flux constraint

Breaks frozen flux constraint, Can balance reconnection electric field

Write in dimensionless variables,

$$\hat{v} = \frac{v}{v_A} \quad \hat{B} = \frac{B}{B_0} \quad \hat{j} = \frac{j}{\mu_0 B_0 / l} \quad \hat{E} = \frac{E}{v_A B_0} \quad \hat{t} = \frac{t}{\tau} \quad \hat{P} = \frac{P}{P_0}$$

then

$$\hat{E} + \hat{v} \times \hat{B} = \frac{1}{S}\vec{j} + \frac{d_i}{l}(\hat{j} \times \hat{B}) - \beta\frac{d_i}{l}(\nabla \cdot \hat{P_e}) - \left(\frac{d_e}{l}\right)^2 \frac{d\hat{j}}{dt}$$

where $d_i = c/\omega_{pi}$ Ion skin depth $\omega_{pi} = \sqrt{ne^2/m\varepsilon_0}$ Ion plasma frequency

For fusion plasmas, d_i ~ 10 cm (large)

For solar corona, d_i ~ 1 m (not large, but significant)

(more later)

Nonlinear MHD effects

Nonlinear mode coupling

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B} - \eta \vec{j})$$

Let $B_k = b_k e^{i(kx - \omega t)}$

Then $$\frac{\partial b_{k1}}{\partial t} = \nabla \times (v_{k2} \times b_{k3}),$$ where $k_1 = k_2 + k_3$

Energy flows between modes that satisfy the 3-wave sum rule,

Important when there are multiple, nearby reconnection sites,

Important in turbulent reconnection

Generation of mean quantities

Define $<x> = x_{k=0}$ mean quantity, spatially uniform

$$\frac{\partial <B>}{\partial t} = \nabla \times (v_{k1} \times b_{k1}),$$

Mean field generation by reconnection (dynamo)

$$\rho \frac{\partial <v>}{\partial t} = \nabla \times (j_{k1} \times b_{k1}) - \rho (v_{k1} \cdot \nabla v_{k1})$$

Mean flow generation by reconnection (dynamo)

(more later)

In summary

- Above discussion establishes a basic picture of reconnection

- More to come:

  Advances in reconnection dynamics (e.g., plasmoids)
  Huge effects of reconnection on plasma

  (transport, magnetic chaos, acceleration and heating, dynamo)

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Fatima Ebrahimi and Stewart Prager

Lecture plan

1. Reconnection basics	SP
2. Dynamo basics	FE
3. Application to lab plasmas	SP
4. Application to space/astrophysical plasmas	FE
5. Challenges and open questions	FE (probably)